How to apply Topological Data Analysis to the SSH models

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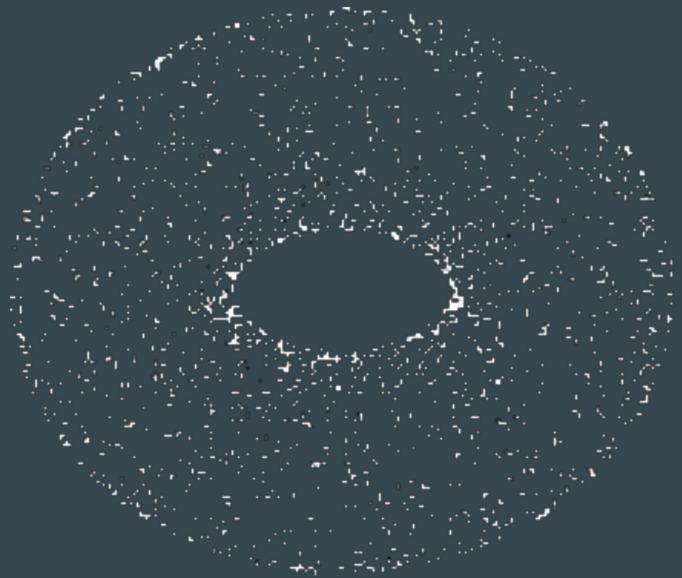
Intro to TDA: Basic homology classes

- n-dimensional homology “counts the number of n-dimensional holes in the topological space”.

  - 0-homology: number of disconnected components
  - 1-homology: number of circles
  - 2-homology: number of balls

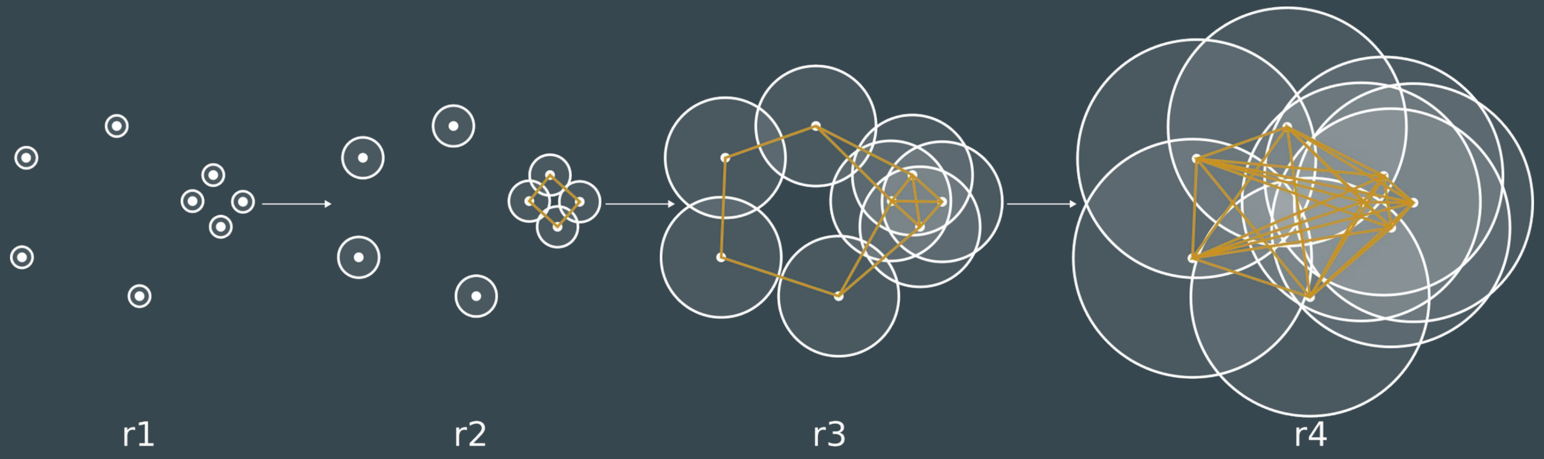
https://www.mobilitysmart.co.uk/two-handled-ceramic-mug.html;
https://www.davidjones.com/product/david-jones-coastal-coffee-mug-20676190;https://www.math.colostate.edu/~adams/teaching/dsci475spr2021/

Intro to TDA: Persistence Homology

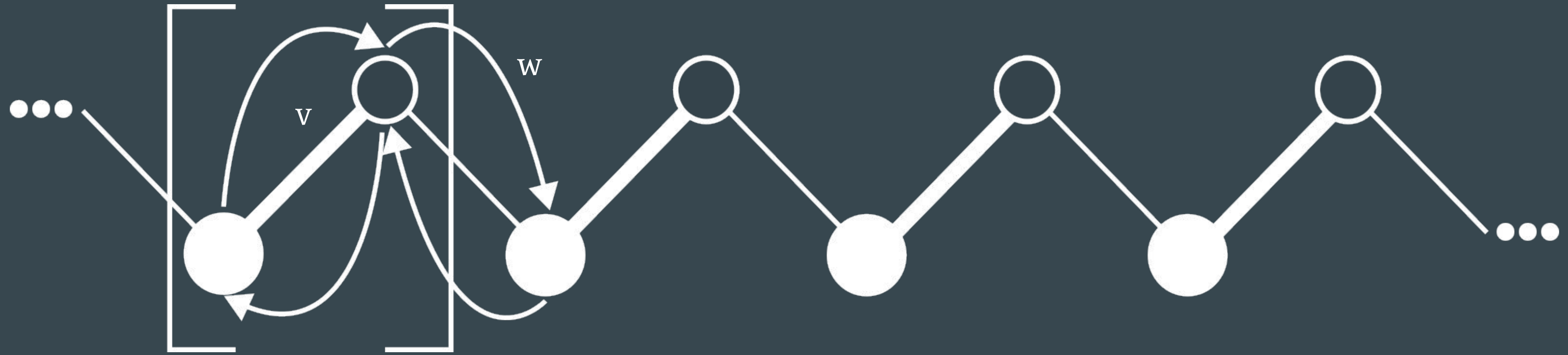


Duan, C., Chen, S., Tian, D., Moura, J. and Kovacevic, J., 2019. Deep Graph Topology Learning for 3D Point Cloud Reconstruction.
He, Y., Huska, M., Kang, S.H. and Liu, H., 2021. Fast algorithms for surface reconstruction from point cloud. In Mathematical Methods in Image Processing and Inverse Problems: IPIP 2018, Beijing, China, April 21–24 (pp. 61-80). Springer Singapore.

Intro to TDA: Persistence Homology

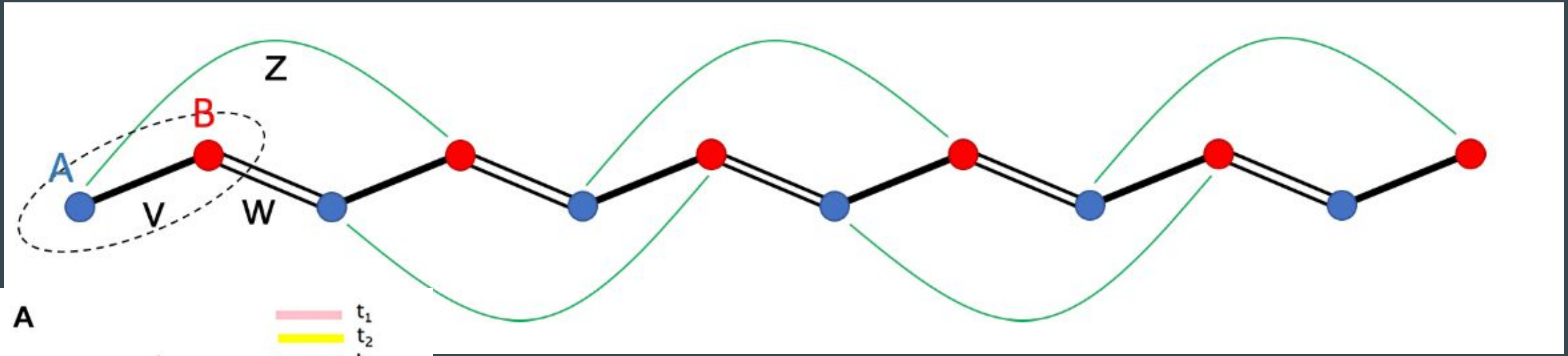


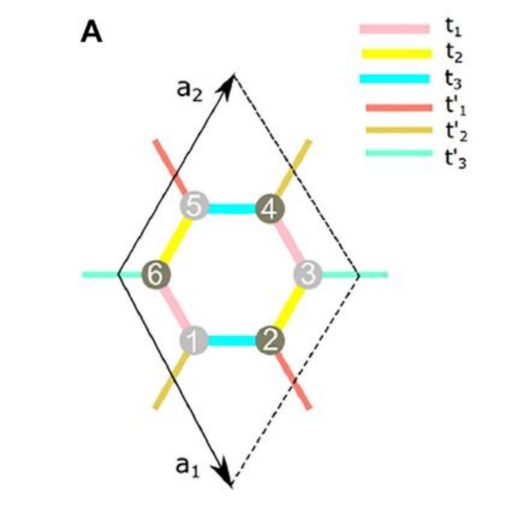
The Su-Schrieffer-Heeger (SSH) Model

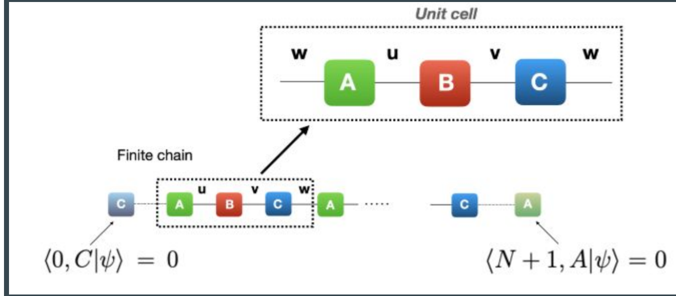


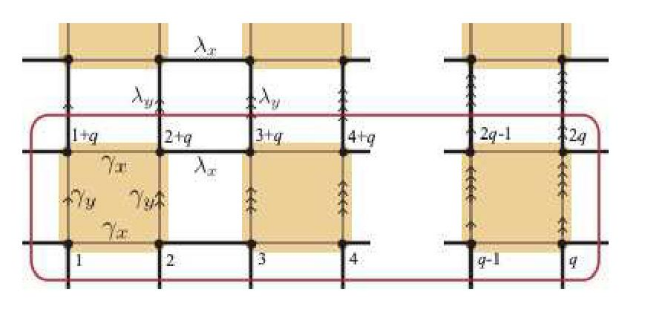
$$H = \sum_{n=1}^{N} (v_n c_{n,1}^{\dagger} c_{n,2} + w_n c_{n,2}^{\dagger} c_{n+1,1} + h.c.)$$

The family of Su-Schrieffer-Heeger (SSH) Models









Liu, Q., Wang, K., Dai, J.X. and Zhao, Y.X., 2022. Takagi Topological Insulator on the Honeycomb Lattice. arXiv preprint arXiv:2205.05873; Otaki, Y. and Fukui, T., 2019. Higher-order topological insulators in a magnetic field. Physical Review B, 100(24), p.245108; Li, C. and Miroshnichenko, A.E., 2018. Extended ssh model: Non-local couplings and non-monotonous edge states. Physics, 1(1), pp.2-16. Anastasiadis, A., Styliaris, G., Chaunsali, R., Theocharis, G. and Diakonos, F.K., 2022. Bulk-edge correspondence in the trimer Su-Schrieffer-Heeger model. arXiv preprint arXiv:2202.13789.

The Goal:

To create a package that calculates a topological phase diagram for any arbitrary dimensional SSH model

Improvements upon existing packages:

1. More user-friendly – easy to run on different models
2. Optimise several functionalities
3. Test and extend to more complicated models

Focus on:

The general flow of the package and challenges in the extension to more models

Park, S., Hwang, Y. and Yang, B.J., 2022. Unsupervised learning of topological phase diagram using topological data analysis. Physical Review B, 105(19), p.195115.

How does topology work in the SSH models

Some robust property that is invariant under smooth deformation of the system.

↓

Find the underlying topology in the system, homology classes? knots and braids?

↙ ↘

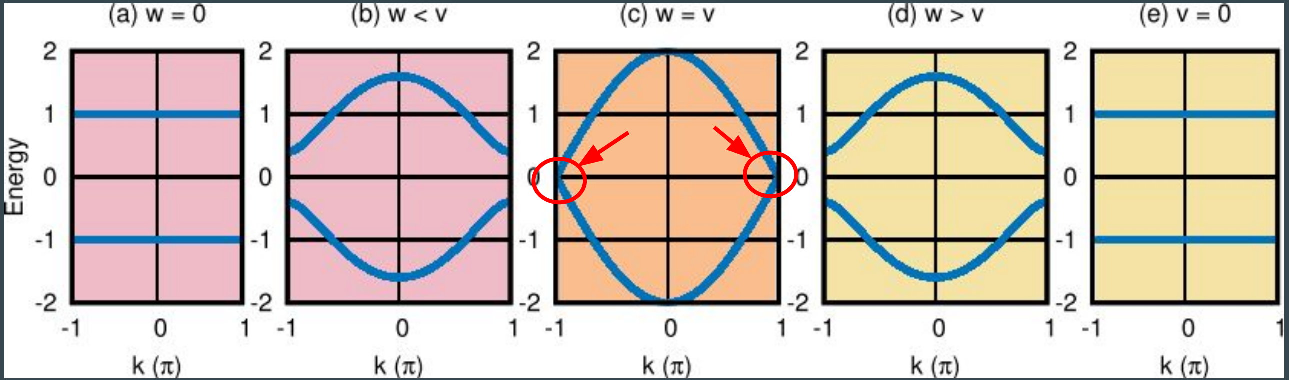
Calculate the topological invariant analytically

Topological Data Analysis

Machine learning

Topology in the Hermitian Chiral SSH models: an example

$$H = \sum_{n=1}^{N} (v_n c_{n,1}^\dagger c_{n,2} + w_n c_{n,2}^\dagger c_{n+1,1} + h.c.) \longrightarrow H(k) = \begin{pmatrix} 0 & v + e^{ik} w \\ v + e^{-ik} w & 0 \end{pmatrix}$$



Invariant in the system:

Presence of robust edge states.

Definition of smooth deformation:

From system A to system B, the transformation is a smooth deformation if and only if the band gap stays open during the transformation.

Batra, N. and Sheet, G., 2019. Understanding Basic Concepts of Topological Insulators Through Su-Schrieffer-Heeger (SSH) Model. arXiv preprint arXiv:1906.08435.

The topological invariant: the Berry phase

We define a principal line bundle where the k-space: (-π, π) is the base manifold and the corresponding nth eigenvector of the bloch matrix is the fiber for each k.

The Berry phase determines the holonomy element of the connection defined on the Hermitian Hilbert space, which has been shown to be quantized in chiral systems:

$$\gamma = -\frac{1}{\pi}\int_c i\langle\psi(k)|\frac{d}{dk}|\psi(k)\rangle$$

$$\begin{cases} 1, & \text{if } v < w. \\ 0, & \text{if } v > w. \\ \text{undefined}, & \text{if } v = w. \end{cases}$$

Topological data analysis on the eigenspace

$$k: (-\pi, \pi) \longrightarrow RP^1 \cong S^1$$

v < w

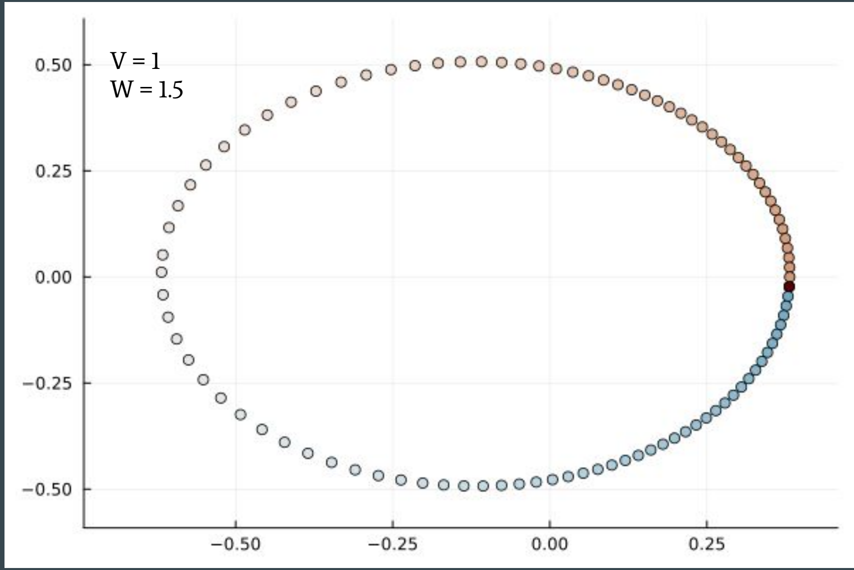
$$d_{HS}(\psi_1, \psi_2) = \sqrt{1 - |\langle \psi_1 | \psi_2 \rangle|^2}$$

v > w

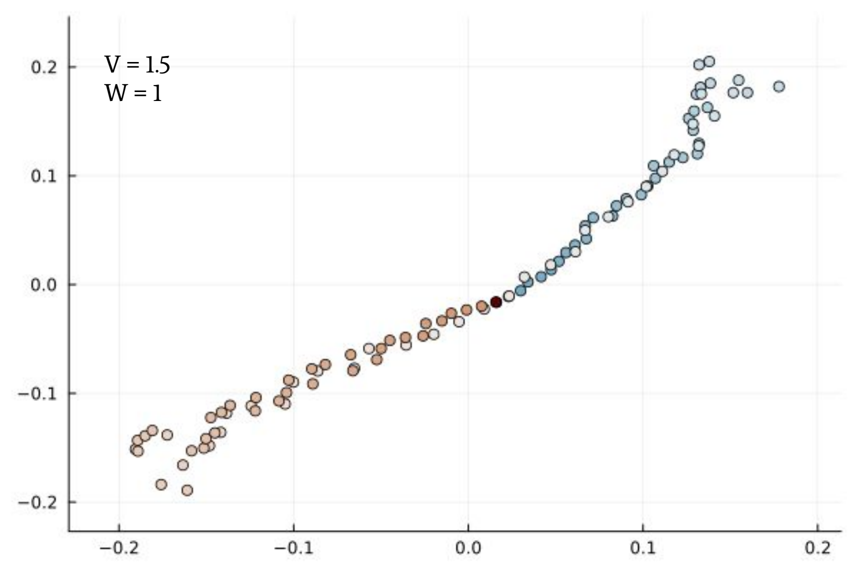
The problem is now simplified to searching for the circle in the eigenspace!

Step 1, Observe the states in the eigenspace via MDS

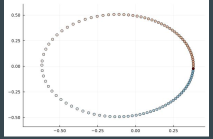
v < w



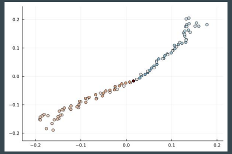
v > w



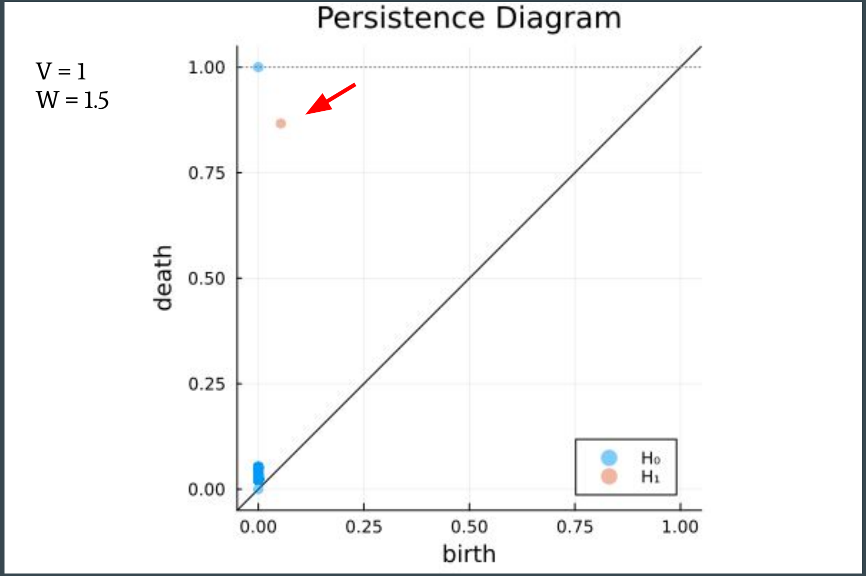
Step 2, Compute the persistence diagrams for all pairs of (v,w)

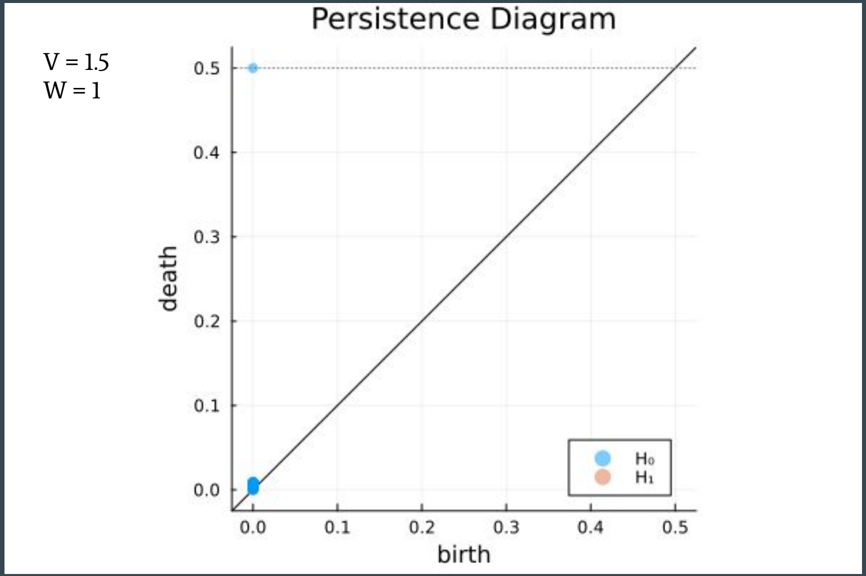


$v < w$

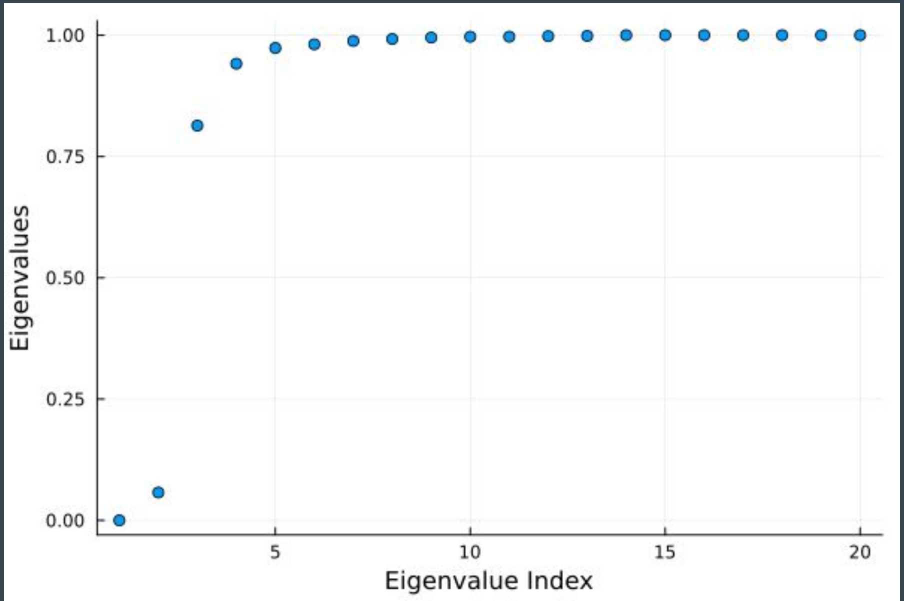


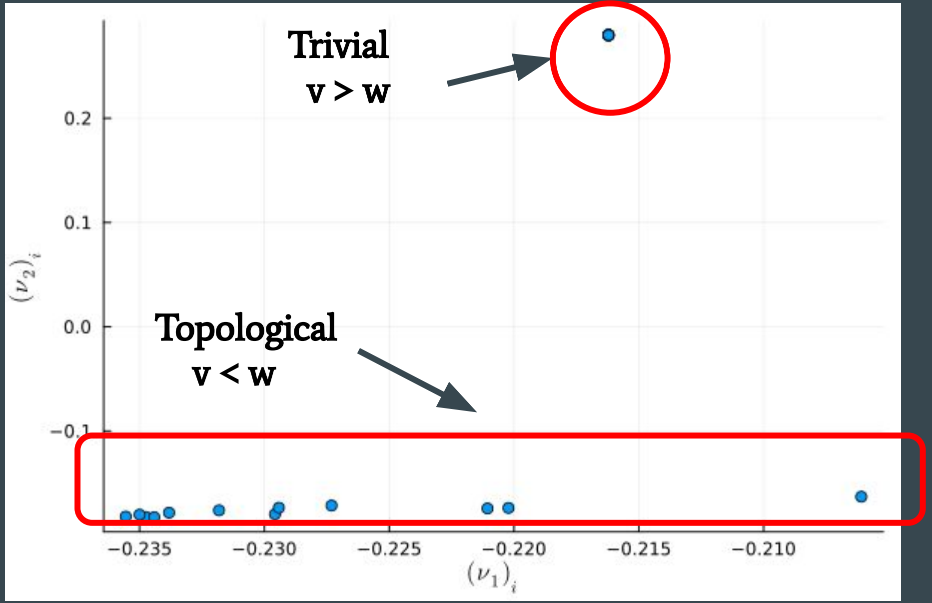
$v > w$





Step 3, produce a random walk matrix out of persistence diagrams





v < w Topological

v > w Trivial

Now, what about non-Hermitian or non-chiral systems?

No bulk-boundary correspondence guaranteed!

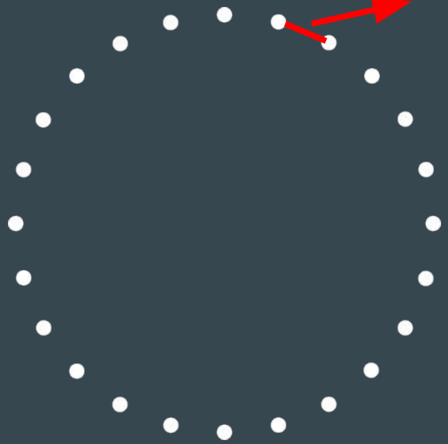
$$H = \sum_{n=1}^{N} (v_n c_{n,1}^{\dagger} c_{n,2} + w_n c_{n,2}^{\dagger} c_{n+1,1} + h.c.)$$

$$H(k) = \begin{pmatrix} 0 & v + e^{ik} w \\ v + e^{-ik} w & 0 \end{pmatrix}$$

Analytical invariant

TDA

$$\gamma = -\frac{1}{\pi} \int_C \langle \psi(k) | \frac{d}{dk} | \psi(k) \rangle$$



Need a new distance metric

And what about non-chiral systems....

Conclusion

Using TDA to study the SSH family

Hermitian

Test case: original SSH done

1. More test cases: higher dimensional, non-chiral
2. More optimisation
3. Why did it work, and how to extend it to others

Non-Hermitian

1. Non-bloch band theory
2. New metric for TDA
3. ...

Thanks for listening, questions?